Name ____ Class _______________ Date _______________

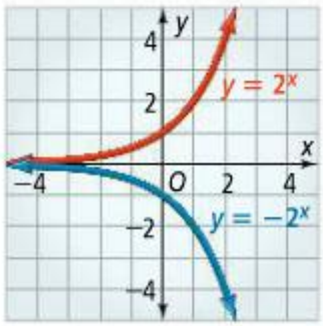
7-6 Exponential Functions

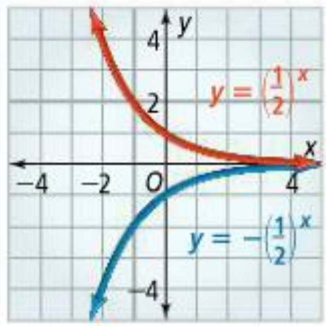
Functions that can be modeled by an equation of the form $y = a \cdot b^x$ are exponential functions. These functions have properties that are different from the properties of linear and quadratic functions.

Definition

An **exponential function** is a function of the form $y = a \cdot b^x$, where $a \neq 0$, $b > 0$, $b \neq 1$, and x is a real number.

Examples





Consider the three function tables below.

x	y
0	3
1	9
2	27
3	81
4	243
5	729

x	y
0	3
1	6
2	12
3	24
4	48
5	96

x	y
0	2
1	10
2	50
3	250
4	1250
5	6250

EXAMPLE

1. For each of the tables on the previous page, extend them two units in each direction. Use the common difference in the *x*-values and the common ratio in the *y*-values to do the extension. The first table is done for you.

x	y
−2	$\frac{1}{3}$
−1	1
0	3
1	9
2	27
3	81
4	243
5	729
6	2187
7	6561

2. Plot the points in each of your extended tables on separate coordinate grids. Connect the points with a smooth curve. The domain of each function is all real numbers and that the range is all positive real numbers. Explain why there are negative values for *x* but not for *y*.

3. For each of the tables, identify the starting value *a* and the common ratio *b*. For the first table, *a* is 1 and *b* is 3. Next, write the exponential function that describes each table. The function for the first table is $f(x) = 1 \bullet 3x$. Check if your function is correct by substituting in *x*-values and seeing if the function produces values for *y* that match the values in the table.

YOUR TURN

Determine whether each table or rule represents a linear or an exponential function. Explain.

1.

x	1	2	3	4
y	3	9	27	81

2.

				4
				21

3. $y = 5 \bullet 2^x$

4. $y = 6 \bullet x^3$

5. $y = 3x - 8$

6. $y = 4 \bullet 0.3^x$

Evaluate each function for the given value.

7. $f(x) = 5^x$ for $x = 4$

8. $h(t) = 3 \bullet 4^t$ for $t = -3$

9. $y = 8 \bullet 0.7^x$ for $x = 3$

Graph each exponential function.

10. $f(x) = 3^x$

11. $y = 0.25^x$

12. $y = 8 \bullet 1.2^x$

13. What is the solution or solutions of $3^x = 5x$?

14. An investment of \$8000 in a certain Certificate of Deposit (CD) doubles in value every seven years. The function that models the growth of this investment is $f(x) = 8000 \cdot 2^x$, where x is the number of doubling periods. If the investor does not withdraw any money from this CD, how much money will be available for withdrawal after 28 years?

15. A population of amoebas in a petri dish will triple in size every 20 minutes. At the start of an experiment the population is 800. The function $y = 800 \cdot 3^x$, where x is the number of 20 minute periods, models the population growth. How many amoebas are in the petri dish after 3 hours?

16. A new car costs \$15,000 to build in 2010. The company's financial analysts expect costs to rise by 6% per year for the 10 years they are planning to build the car. The cost to build the car can be modeled by the function $f(t) = 15{,}000\,(1.06)^t$, where t is the number of years after 2010. How much will it cost the company to build the car in 2017?

Evaluate each function over the domain {–2, –1, 0, 1, 2, 3}. As the values of the domain increase, do the values of the range *increase* or *decrease*?

17. $f(x) = 3^x$

18. $y = 4.2^x$

Which function has the greater value for the given value of x?

23. $y = 5^x$ or $y = x^5$ for $x = 2$

Solve each equation.

25. $3^x = 81$

27. $4^x + 4 = 68$

29. **Reasoning** The function that models the growth of a $1000 investment that earns 7% per year is $f(x) = 1000(1.07)^x$. How do you think you would write a function that models the growth of $1500 that earns 8% per year? Use that function to determine how much money a person would have after 5 years if she invested $1500 in an account earning 8% per year.

30. **Writing** Discuss the differences between exponential functions with a base of 2 and 3, $y = 2^x$ and $y = 3^x$, and quadratic and cubic functions $y = x^2$ and $y = x^3$. Focus on the shapes of the different graphs and rates of growth.

31. **Open-Ended** Find the value of each of the functions a) $f(x) = 2x^2$ and b) $f(x) = 2 \bullet 2^x$ for $x = 5$. Write another quadratic function and another exponential function with a base of two whose values at $x = 5$ are between the values you found for functions a and b.