Name ____ Class _______________ Date ________________

7-7 Exponential Growth and Decay

Exponential functions can model the growth or decay of an initial amount.

The basic exponential function is $y = a \bullet b^x$ where

a represents the initial amount

b represents the growth (or decay) factor. The growth factor equals 100% plus the percent rate of change. The decay factor equals 100% minus the percent rate of decay.

x represents the number of times the growth or decay factor is applied.

y represents the result of applying the growth or decay factor *x* times.

Problem

A gym currently has 2000 members. It expects to grow 12% per year. How many members will it have in 6 years?

In Exercises 1–3, identify *a, b,* and *x*. Then use them to write the exponential function that models each situation. Finally use the function to answer the question.

1. When a new baby is born to the Johnsons, the family decides to invest $5000 in an account that earns 7% interest as a way to start the baby's college fund. If they do not touch that investment for 18 years, how much will there be in the college fund?

2. The local animal rescue league is trying to reduce the number of stray dogs in the county. They estimate that there are currently 400 stray dogs and that through their efforts they can place about 8% of the animals each month. How many stray dogs will remain in the county 12 months after the animal control effort has started?

3. A basket of groceries costs $96.50. Assuming an inflation rate of 1.8% per year, how much will that same basket of groceries cost in 20 years?

Exponential Growth and Decay

While it is usually fairly straightforward to determine *a*, the initial value, you often need to read the problem carefully to make sure that you are correctly identifying *b* and *x*. This is especially true when considering situations where the given growth rate is applied in intervals that are not the same as the given value of *x*.

$A = P\left(1+\frac{r}{n}\right)^{nt}$ where *A* is the final balance, *P* is the initial deposit, *r* is the annual interest rate, *n* is the number of times the interest is compounded per year and *t* is the number of years.

Problem

In Exercises 4 and 5, identify *a, b,* and *x*. Then write the exponential function that models the situation. Finally, use the function to answer the question.

3. You invest $2000 in an investment that earns 6% interest, compounded quarterly. How much will the investment be worth after 5 years?

4. You invest $3000 in an investment that earns 5% interest, compounded monthly. How much will the investment be worth after 8 years?

6. The formula that financial managers and accountants use to determine the value of investments that are subject to compounding interest is

Identify the initial amount *a* and the growth factor *b* in each exponential function. (Hint: In the exponential equation $y = a \cdot b^x$, *a* is the initial amount and *b* is the growth factor when $b > 1$.)

1. $f(x) = 2 \cdot 3^x$

2. $y = 5 \cdot 1.06^x$

2. $g(t) = 6^t$

4. $h(x) = -3 \cdot 2^x$

Use the given function to find the balance in each account after the given period.

5. $3000 principal earning 4% compounded annually, after 6 years
   $f(x) = 3000 \cdot (1.04)^6$

6. $2000 principal earning 6.8% compounded annually, after 3 years
   $f(x) = 2000 \cdot (1.068)^3$

Find the balance in each account after the given period.

7. $5000 principal earning 4% compounded annually, after 10 years

8. \$3500 principal earning 3.6% compounded annually, after 2 years

15. The town manager reports that incoming revenues for a given year were \$2 million. The budget director predicts that revenues will increase by 4% per year. How much revenue will the town have available 10 years from the date of the town manager's report if the equation that models the growth is $f(x) = 2{,}000{,}000 \cdot (1.04)^x$?

16. A fisheries manager determines that there are approximately 3000 bass in a lake.

a. The population is growing at a rate of 2% per year. The function that models that growth is $y = 3000 \cdot 1.02^x$. How many bass will live in the lake after 4 years?

b. How many bass will live in the lake after 7 years?

c. About how long will it be before there are 4000 bass in the lake?