Name ______________________________________ Class ______________ Date _______________

7-8 Geometric Sequences

A geometric sequence is a sequence in which the ratio between consecutive terms is constant. This ratio is called the common ratio. Every geometric sequence has a starting value and a common ratio.

General form

Key Concept Geometric Sequence

A geometric sequence with a *starting value* a and a *common ratio* r is a sequence of the form

$a, ar, ar^2, ar^3, \ldots$

A *recursive definition* for the sequence has two parts:

$a_1 = a$	Initial condition
$a_n = a_{n-1} \cdot r$, for $n \geq 2$	Recursive formula

An *explicit definition* for this sequence is a single formula:

$a_n = a_1 \cdot r^{n-1}$, for $n \geq 1$

Every geometric sequence has a starting value and a common ratio. The starting value and common ratio define a unique geometric sequence.

Problem

Is the following a geometric sequence? 1, 2, 4, 8, 16, …

$$1 \xrightarrow{\times 2} 2 \xrightarrow{\times 2} 4 \xrightarrow{\times 2} 8 \xrightarrow{\times 2} 16$$

Determine whether the sequence is a geometric sequence. Explain.

1. 5, 10, 20, 40, …

2. 3, 9, 27, 81, …

3. 1, 4, 9, 16, …

4. 24, 20, 16, 12, …

5. 25, 20, 15, 10, …

6. –48, 96, –192, 384, …

Any geometric sequence can be defined by both an explicit and a recursive definition. The recursive definition is useful for finding the next term in the sequence.

$$a_1 = a;\ a_n = a_{n-1} \cdot r$$

In this formula,

- a_1 represents the first term
- a_n represents the nth term
- n represents the term number
- r represents the common ratio
- a_{n-1} represents the term immediately before the nth term

Problem

What is a recursive formula for the geometric sequence 3, 12, 48, 192, 768, …?

Write the recursive formula for each geometric sequence.

7. 5, 25, 125, 625, …

8. –2, 6, –18, 54, …

9. 96, 72, 54, 40.5, …

10. 10, –10, 10, –10, …

An explicit formula is more convenient when finding the *n*th term.

Problem

What is an explicit formula for the geometric sequence –2, 2, –2, 2, …?

$a_n = a_1 \cdot r^{n-1}$ Use the explicit definition for a geometric sequence.

$a_n = -2 \cdot (-1)^{n-1}$ Replace a_1 with –2 and r with –1.

Write the explicit formula for each geometric sequence.

11. –3, 3, –3, 3, …

12. 1, 0.5, 0.25, 0.125, …

13. 200, 100, 50, 25, …

14. 6, 12, 24, 48, …

Use the list below to complete the diagram.

- $a_n = a_{n-1} \cdot r$, n ≥2
- sequence in which the ratio of any term to its preceding term is constant
- a in $a_n = a_{n-1} \cdot r$, n ≥2
- r in $a_n = a_{n-1} \cdot r$, n ≥2
- explicit definition for a sequence
- sequence in which the difference between every pair of consecutive terms is the same

$a_n = a_1 \cdot r^{n-1}$ for $n \geq 1$

arithmetic sequence

geometric sequence

recursive definition for a geometric sequence

common ratio

initial value

Identify each sequence as arithmetic, geometric, or neither.

15. 1, 6, 11, 16, …

16. 125, 62.5, 31.25, 15.625 …

17. 1, 5, 10, 16, …

18. –5, 5, –5, 5, …

19. **Biology** A certain population of finches is decreasing by 6% every year. The current number of finches in the population is 456. Write the explicit and recursive formulas for the geometric sequence formed by the decrease in the number of finches.

20. **Compare and Contrast** Explain how a geometric sequence and an arithmetic sequence are the same. How are they different?

21. A geometric sequence is represented by the function $f(x) = 3 \cdot 2^{x-1}$. What is the initial value of the sequence and the common ratio? Find the first 4 terms of the sequence.